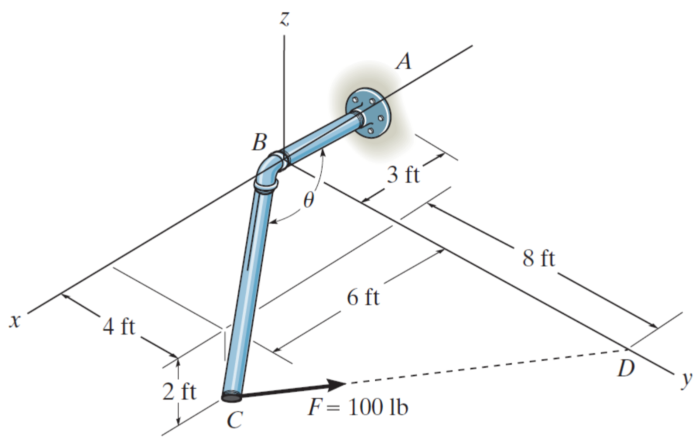
Problem 2-126

Determine the magnitude of the projected component of the 100-lb force acting along the axis BC of the pipe.



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Solution

Write the position vectors to the points B, C, and D.

$$\mathbf{r}_B = \langle 0, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 6, 4, -2 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle 0, 12, 0 \rangle \text{ ft}$$

The unit vector going from C to B is

$$\hat{\mathbf{u}}_{CB} = \frac{\mathbf{r}_B - \mathbf{r}_C}{|\mathbf{r}_B - \mathbf{r}_C|} = \frac{\langle -6, -4, 2 \rangle}{\sqrt{(-6)^2 + (-4)^2 + (2)^2}},$$

and the unit vector going from C to D is

$$\hat{\mathbf{u}}_{CD} = \frac{\mathbf{r}_D - \mathbf{r}_C}{|\mathbf{r}_D - \mathbf{r}_C|} = \frac{\langle -6, 8, 2 \rangle}{\sqrt{(-6)^2 + (8)^2 + (2)^2}}.$$

The force is then

$$\mathbf{F} = F\hat{\mathbf{u}}_{CD} = 100 \frac{\langle -6, 8, 2 \rangle}{\sqrt{(-6)^2 + (8)^2 + (2)^2}} \text{ lb}.$$

Take the dot product of $\mathbf{F}$ with $\hat{\mathbf{u}}_{CB}$ to get the component of the force along the pipe's axis.

$$\mathbf{F} \cdot \hat{\mathbf{u}}_{CB} = 100 \frac{\langle -6, 8, 2 \rangle}{\sqrt{(-6)^2 + (8)^2 + (2)^2}} \cdot \frac{\langle -6, -4, 2 \rangle}{\sqrt{(-6)^2 + (-4)^2 + (2)^2}} \text{ lb} = \frac{100}{\sqrt{91}} \text{ lb}$$

Therefore, the magnitude of this component is

$$|\mathbf{F} \cdot \hat{\mathbf{u}}_{CB}| = \frac{100}{\sqrt{91}} \text{ lb} \approx 10.5 \text{ lb}.$$